Week 5: Midterm revision session

Jack Blumenau & Philipp Broniecki

University College London

Introduction to Quantitative Methods

1. Administrative information
2. Answer advice
3. Hypothesis testing
4. Simple linear regression
5. Multiple linear regression

Overview

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Administrative information

- Midterm will be released at 2pm on November 3rd
- Midterm is due at 2pm on November 8th
- All submissions via Turnitin
- Usual late penalties apply
- Usual extenuating circumstances policies apply

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  - Substantive information. e.g. What does this tell us about our research question?

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- Answer the question! If you are asked to answer a policy relevant question, you should not simply report a p-value without commenting on the substance.
- You can use R to answer any question where you think it might be useful. But if the question tells you to 'show your work', that means you need to show that you know how the values from R were calculated!

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2. Are the means in subgroups of our data different? E.g., is average income in Scotland different from income in Wales?
3. Is effect of some X variable on some Y variable different from 0?

Hypothesis test sequence

- State the hypothesis and the null hypothesis
- Calculate a test-statistic
- Derive the sampling distribution of the test statistic under the assumption that the null hypothesis is true
- Calculate the p-value
- State a conclusion

Test for the sample mean: hypothesis

Is the die loaded

Each outcome on a die is equally likely. Thus, the average outcome from throwing a fair die often is 3.5. If we take a die and throw it 100 times and and get an average of 3.46, is that evidence for a loaded die or not?

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- Null Hypothesis: die is fair. The small difference we find is due to chance.
- Hypothesis: The die is loaded. The difference is systematic

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- The t-statistic is the difference in means. It's units are average distances from the true mean (standard deviations).
- We do not know the standard deviation of the sampling distribution, so we estimate it with the standard error

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- Formally, it is an *estimate* for the standard deviation of the *sampling distribution*

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- Suppose: $\sigma_Y = 1.69$

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  - 68 sample means will be within 1 standard error of true mean
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- We therefore know that the null is not that unlikely $\rightarrow$ We fail to reject the null hypothesis

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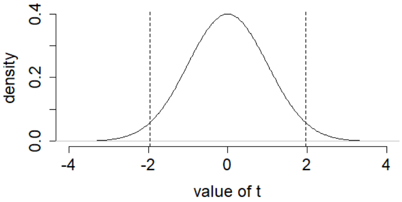

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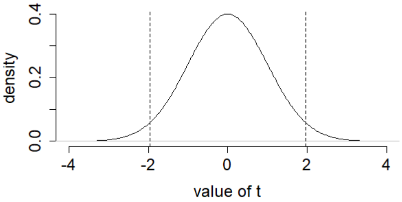


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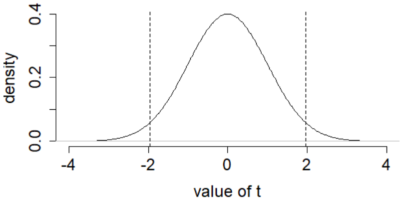


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2*(1 - pnorm(0.24))
[1] 0.8103303
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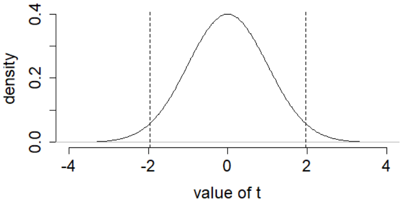

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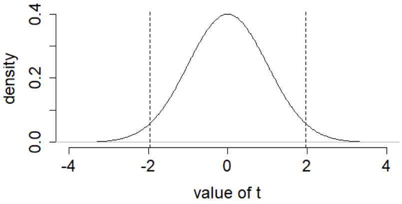


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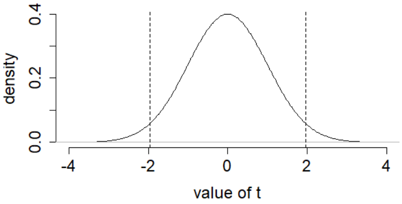


```
## probability that
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2*(1 - pt(0.24, df = 99))
[1] 0.8108265
```

Test for the sample mean: R

- You can carry out the individual steps or you can use the t.test() function

```
t.test( var.name, mu = value of H0 , conf = 0.95 )
```

t-tests for the difference in two means

- Often we are interested in whether the mean for one group is different from the mean for another group
    - Is woman's income different to men's income?
    - Do Democratic and Republican senators receive different amounts of campaign donations?

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  - Is woman's income different to men's income?
  - Do Democratic and Republican senators receive different amounts of campaign donations?
- t-tests can also be used to compare the means of two groups
- Requires an interval-level dependent variable (Y) and binary independent variable (X)

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  - The variance (s^2_Y) is just the standard deviation (s_Y) squared
- $n_{X=0}$ and $n_{X=1}$ are the number of observations for each group

Test for the difference in means: critical value of t

- Assuming that sample size is large (> 30), the critical t value is 1.96
- To know the exact critical value, we need to know the degrees of freedom (df)
- You could do it in R using the t.test() function which computes the correct number of df for you

Test for the difference in means: p value

- Once we know the correct t value, getting the p value is the same as in the t-test for the sample mean if the sample is large
- If the sample is small, use R's t.test() function

Test for the difference in means: R

- You need a continuous dependent variable (DV)
- A binary independent variable (IV)
- Unless stated otherwise, the null is usually there is no difference in means. Hence, mu $= 0$

```
t.test(DV ~ IV, mu = 0, conf = 0.95)
```

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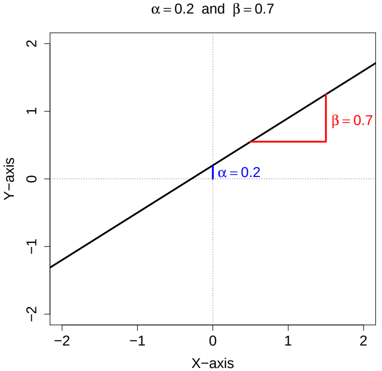
Simple linear regression: intuition

- How are two phenomena (X and Y) related?

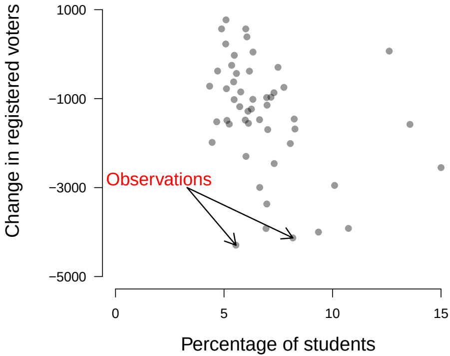
Linear relationships

- The most straightforward way of describing the relationship between two variables is with a line
- A line can be represented by this expression: $Y = \alpha + \beta X$



- α is the intercept: the value of Y where $X = 0$
- β is the slope: the amount that Y increases when X increases by one unit
- Here, a one-unit increase in X is associated with a 0.7-unit increase in Y

The linear regression line



- Observations $i = 1, \ldots, n$

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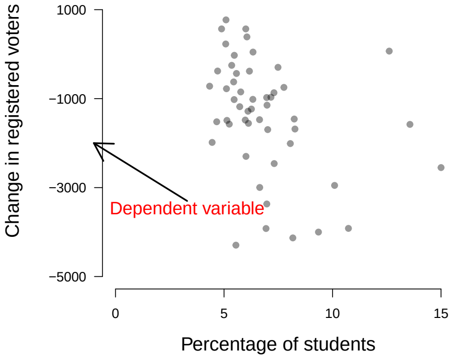

- Observations $i = 1, \ldots, n$
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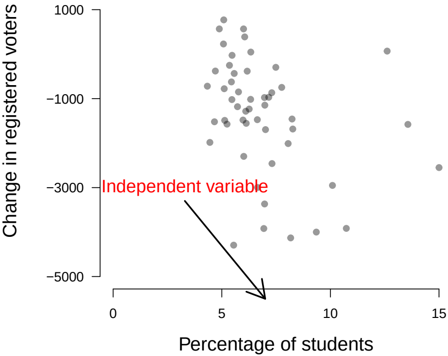

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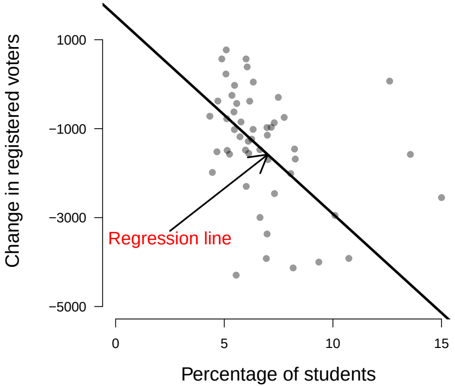

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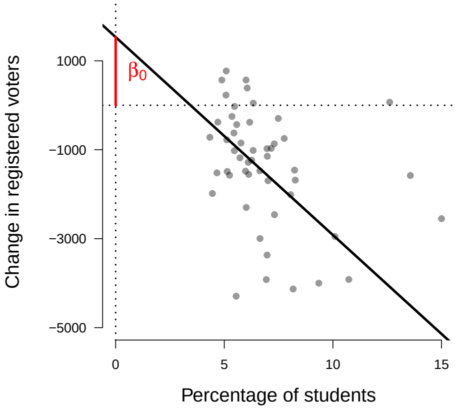

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- The regression line.
- β_0 is the intercept.

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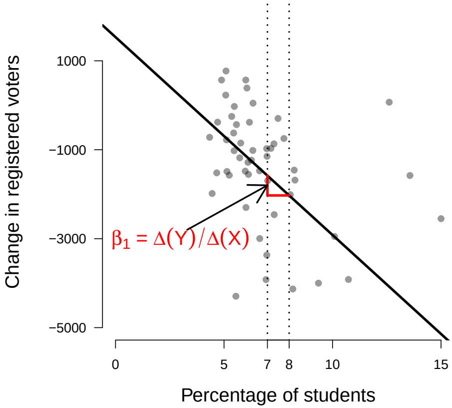

- Observations $i = 1, \dots, n$
- Y is the dependent variable.
- X is the independent variable.
- The regression line.
- β_0 is the intercept.
- β_1 is the slope.

Application to voter registration

- For the regression of registration on the percentage of students we obtain:

DV: Δvoters	$\hat{\beta}_k$, $(\hat{\sigma}_{\hat{\beta}_k})$
(Intercept)	1532.69
	(192.41)
students	-444.97
	(26.99)
R^2	0.32
N.	573

where the numbers in brackets are the standard errors of the coefficients.

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- To test the government's hypothesis:

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R^2	0.32
N.	573

- To test the government's hypothesis:

$$t = \frac{\hat{\beta}_1 - \beta_{H_0}}{\hat{\sigma}_{\hat{\beta}_1}}$$

Application to voter registration

DV: Δvoters	$\hat{\beta}_k$, $(\hat{\sigma}_{\hat{\beta}_k})$
(Intercept)	1532.69
	(192.41)
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- To test the government's hypothesis:

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- To test the government's hypothesis:

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- Can we reject the null hypothesis at $\alpha = 0.05$?

Application to voter registration

$$t = \frac{\hat{\beta}_1 - \beta_{H_0}}{\hat{\sigma}_{\hat{\beta}_1}} = \frac{-445 - 0}{27} \approx -16.48$$

- The probability of observing a value of the t-statistic outside the interval $[-1.96, 1.96]$ is less than five percent under the standard normal distribution.
- As the t-statistic is clearly outside this interval, the probability that H_0 is correct is less than five percent.
- We can therefore reject the government's claim at the five percent significance level.

Application to voter registration

R will automatically calculate the correct test-statistic for you:

```
summary(my_linear_model)

Residuals:
    Min      1Q  Median      3Q     Max
-5163.4  -787.0   -21.7   924.5  4921.4

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  1532.69     192.41   7.966 8.93e-15 ***
students     -444.97      26.99 -16.489  < 2e-16 ***
---
Signif. codes:  0 ?***? 0.001 ?**? 0.01 ?*? 0.05 ?.? 0.1 ? ? 1

Residual standard error: 1525 on 571 degrees of freedom
Multiple R-squared:  0.3226,Adjusted R-squared:  0.3214
F-statistic: 271.9 on 1 and 571 DF,  p-value: < 2.2e-16
```

Overview

Multiple linear regression: intuition

- We can control for confounders with multiple linear regression

More than two independent variables

```
## Specify the model with 3 independent variables
linear_model_3 <- lm(AfD ~ christian + east
+ migrantfraction , data = results)

## Output in a nice format
screenreg(list(linear_model_1, linear_model_2, linear_model_3))

===================================================
                 Model 1      Model 2      Model 3
---------------------------------------------------
(Intercept)        21.29 ***     7.82 ***   11.78 ***
                   (0.76)       (1.30)      (1.90)
christian          -0.16 ***     0.03        0.00
                   (0.01)       (0.02)      (0.02)
eastTRUE                        11.77 ***    9.14 ***
                                (0.99)      (1.35)
migrantfraction                             -0.09 **
                                            (0.03)
---------------------------------------------------
R^2                 0.36         0.56        0.58
Adj. R^2            0.35         0.56        0.57
Num. obs.         299          299         299
===================================================
*** p < 0.001, ** p < 0.01, * p < 0.05
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- The coefficient on `migrantfraction` ($\hat{\beta}_3$) is negative and significant

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- The coefficient on `migrantfraction` ($\hat{\beta}_3$) is negative and significant
- The coefficient on `east` ($\hat{\beta}_2$) is smaller in model 3

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```

- The coefficient on `migrantfraction` ($\hat{\beta}_3$) is negative and significant
- The coefficient on east ($\hat{\beta}_2$) is smaller in model 3
- The R^2 has increased